



Subject card

Subject name and code	Fraktals, PG_00021049						
Field of study	Mathematics						
Date of commencement of studies	October 2022	Academic year of realisation of subject			2022/2023		
Education level	second-cycle studies	Subject group			Optional subject group Subject group related to scientific research in the field of study		
Mode of study	Full-time studies	Mode of delivery			at the university		
Year of study	1	Language of instruction			Polish		
Semester of study	1	ECTS credits			5.0		
Learning profile	general academic profile	Assessment form			exam		
Conducting unit	Department of Probability Theory and Biomathematics -> Faculty of Applied Physics and Mathematics						
Name and surname of lecturer (lecturers)	Subject supervisor		prof. dr hab. Joanna Janczewska				
	Teachers		prof. dr hab. Joanna Janczewska				
Lesson types and methods of instruction	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	30.0	15.0	0.0	15.0	0.0	60
	E-learning hours included: 0.0						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours		Self-study	SUM
	Number of study hours	60		5.0		60.0	125
Subject objectives	The aim of the lecture is to introduce the central ideas and concepts of fractals as well as many related topics.						
Learning outcomes	Course outcome	Subject outcome			Method of verification		
	K7_U11	A student understands relations between the fractal theory and dynamical systems.			[SU3] Assessment of ability to use knowledge gained from the subject		
	K7_W05	A student can define the space of Hausdorff type.			[SW1] Assessment of factual knowledge		
	K7_K02	A student is able to explain the notion of fractal by the use of examples from biology.			[SK4] Assessment of communication skills, including language correctness		
	K7_U09	A student can apply the knowledge acquired at other lectures.			[SU3] Assessment of ability to use knowledge gained from the subject		
K7_U02	A student knows basic notions and facts of discrete and continuous dynamical systems.			[SU4] Assessment of ability to use methods and tools			
Subject contents	The Banach contraction principle. Examples of fractals. Why do so many people study fractals? Fractal spaces with the Hausdorff metric. Iterated function systems (IFS). A fractal dimension, the Hausdorff dimension and a topological dimension. The Mandelbrot definition of fractals. Julia sets. The Mandelbrot set. Discrete dynamical systems. Continuous dynamical systems. A definition and properties of the Poincare map. Attractors and repellers. The Feigenbaum cascade. The Smale horseshoe - a geometric description. Properties of the invariant set of the Smale horseshoe.						
Prerequisites and co-requisites	Mathematical analysis. Topology. Ordinary differential equations.						
Assessment methods and criteria	Subject passing criteria		Passing threshold		Percentage of the final grade		
	A maths test		50.0%		50.0%		
	Project (MAX 5 students in each group).		100.0%		50.0%		

Recommended reading	Basic literature	<p>1. Jacek Kudrewicz, Fraktale i Chaos, Wydawnictwa Naukowo-Techniczne, Warszawa, 2007.</p> <p>2. Lawrence Perko, Differential Equations and Dynamical Systems, Springer, New York, 2001.</p>
	Supplementary literature	<p>1. J.D. Murray, Mathematical Biology. I: An Introduction, Springer-Verlag, New York, 2002.</p> <p>2. H.-O. Peitgen, H. Jurgens, D. Saupe, Chaos and Fractals. New Frontiers of Science, Springer-Verlag, New York, 2004.</p>
	eResources addresses	Adresy na platformie eNauczanie:
Example issues/ example questions/ tasks being completed	<p>1. Is a given subset Z in \mathbb{R}^n compact (connected, nowhere dense)? Justify the answer.</p> <p>2. Calculate the Hausdorff distance between two given subsets A and B in \mathbb{R}^2.</p> <p>3. Calculate a fractal dimension, the Hausdorff dimension and a topological dimension of the Cantor set, the Koch curve, the Sierpiński gasket and carpet.</p> <p>4. Let $w_1, w_2, \dots, w_k: \mathbb{R}^n \rightarrow \mathbb{R}^n$ be given. Prove that $\{\mathbb{R}^n; w_1, w_2, \dots, w_k\}$ is an iterated function system. Calculate the constant of its contraction.</p> <p>5. Solve a linear differential equation of first order $x' = Ax$ in \mathbb{R}^2, where A is a given square matrix 2×2.</p> <p>6. Give a geometric description of the Smale horseshoe map.</p> <p>7. What is it an attractor? Give a short description of the Hénon attractor, the Rössler attractor and the Lorenz attractor.</p>	
Work placement	Not applicable	