



## Subject card

Subject name and code	Chaos theory, PG_00023806						
Field of study	Mathematics						
Date of commencement of studies	October 2022	Academic year of realisation of subject			2023/2024		
Education level	second-cycle studies	Subject group			Optional subject group Subject group related to scientific research in the field of study		
Mode of study	Full-time studies	Mode of delivery			blended-learning		
Year of study	2	Language of instruction			Polish		
Semester of study	3	ECTS credits			4.0		
Learning profile	general academic profile	Assessment form			assessment		
Conducting unit	Department of Probability Theory and Biomathematics -> Faculty of Applied Physics and Mathematics						
Name and surname of lecturer (lecturers)	Subject supervisor		dr hab. Piotr Bartłomiejczyk				
	Teachers		dr hab. Piotr Bartłomiejczyk				
Lesson types and methods of instruction	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	30.0	0.0	0.0	0.0	30.0	60
	E-learning hours included: 30.0						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours		Self-study	SUM
	Number of study hours	60		5.0		35.0	100
Subject objectives	Introduction to advanced methods in studying and description of evolution of systems with trends towards chaotic behaviour. Synthesis of probabilistic, topological and analytical techniques to obtain statistical behaviour.						
Learning outcomes	Course outcome	Subject outcome			Method of verification		
	K7_U10	Finds fixed points, periodic points and dense orbits.			[SU3] Assessment of ability to use knowledge gained from the subject [SU4] Assessment of ability to use methods and tools		
	K7_U06	Ranks dynamical systems. Examines ergodic properties of dynamical systems. Compares the degree of chaos.			[SU2] Assessment of ability to analyse information		
	K7_K04	Interprets chaotic nature of data in their analysis.			[SK5] Assessment of ability to solve problems that arise in practice		
	K7_W03	Finds attractors. Evaluates entropy.			[SW1] Assessment of factual knowledge		
	K7_U09	Evaluates fractal dimension. Finds invariant measures.			[SU1] Assessment of task fulfilment		
Subject contents	<p>LECTURES Revision of selected topics from topology, measure theory and functional analysis. Abstract dynamical systems. Nonlinear contractions and fixed points. Deterministic chaos. Hypercyclicity and linear chaos in Banach spaces. Barnsley operator and attractors. Fractals. Hausdorff measure and dimension. Measurable transformations and invariant measures. Poincare recurrence theorem. Ergodicity. Mixing and weak (mild) mixing. Entropy.</p> <p>SEMINARS Chaotic functions (examples). Bifurcations in the family of logistic maps. Relations between characteristics of trajectories. Sharkovski and Li-Yorke theorems. Barnsley operator. IFS systems. Chaos and Barnsley attractors. Ergodicity. Exactness. Mixing. Evolution of densities. Frobenius-Perron and Koopmann operators. Markov operators on measures. Random dynamical systems. Fractals. Hausdorff dimension. Julia sets. Chaos and fractals on a complex plain.</p>						

Prerequisites and co-requisites	Courses completed: Probability Theory (MAT1013), Functional Analysis II (MAT2003)		
Assessment methods and criteria	Subject passing criteria	Passing threshold	Percentage of the final grade
	Activity	51.0%	10.0%
	Test	51.0%	40.0%
	Research project	51.0%	50.0%
Recommended reading	Basic literature	<ol style="list-style-type: none"> <li>1. S.W. Fomin, I.P. Kornfeld, J.G. Sinaj, Teoria ergodyczna, PWN, Warszawa, 1987.</li> <li>2. A. Lasota and M.C. Mackey, Chaos, Fractals and Noise, Springer, New York, 1994.</li> <li>3. H.O. Peitgen, H. Jurgens, D. Saupe, Granice chaosu. Fraktale, PWN, Warszawa, 1996.</li> <li>4. T.M. Sękowski, Zagadnienia matematycznej teorii chaosu, Wydawnictwo UMCS, Lublin, 2007.</li> </ol>	
	Supplementary literature	<ol style="list-style-type: none"> <li>1. K. Grosse-Erdmann, A.P. Manguillot, Linear Chaos, Springer, 2011.</li> <li>2. W. Szlenk, Wstęp do teorii gładkich układów dynamicznych, BM tom 56, 1982.</li> <li>3. Y. Pesin and V. Climenhaga, Lectures on Fractal Geometry and Dynamical Systems, AMS, Rhode Island, 2009.</li> <li>4. A. Berger, Chaos and Chance, Walter de Gruyter, 2001.</li> <li>5. R. Zaharopol, Invariant Probabilities of Markov-Feller Operators and their Supports, Birkhauser, 2005.</li> <li>6. E.E. Peters, Teoria chaosu a rynki kapitałowe, WIG Press, 1997.</li> <li>7. J. Stachursky, Economis Dynamics: theory and Computation, MIT Press, 2009.</li> <li>8. T. Downarowicz, Entropy in Dynamical Systems, Cambridge University Press, 2011.</li> </ol>	
	eResources addresses	Adresy na platformie eNauczenie: Teoria chaosu 2023/2024 - Moodle ID: 32757 <a href="https://enauczenie.pg.edu.pl/moodle/course/view.php?id=32757">https://enauczenie.pg.edu.pl/moodle/course/view.php?id=32757</a>	
Example issues/ example questions/ tasks being completed	<p>At the beginning of the term the student is provided with the list of topics to be worked out and finally to be presented as a project on a prescribed date. Theoretical components from lectures and seminars are verified on the test. Student's activity on seminars is essential.</p> <p>Find the periodic structure of a dynamical system. Find chaotic features of a dynamical systems. Find an attractor of a dynamical system. Evaluate Hausdorff measure and fractal dimension. Find invariant measures. Investigate ergodicity or mixing of a given transformation.</p>		
Work placement	Not applicable		