Subject card

| Subject name and code | Mathematics, PG_00049709 |  |  |  |  |  |  |
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| Field of study | Management |  |  |  |  |  |  |
| Date of commencement of studies | October 2022 |  | Academic year of realisation of subject |  |  | 2022/2023 |  |
| Education level | first-cycle studies |  | Subject group |  |  | Obligatory subject group in the field of study |  |
| Mode of study | Full-time studies |  | Mode of delivery |  |  | at the university |  |
| Year of study | 1 |  | Language of instruction |  |  | English |  |
| Semester of study | 1 |  | ECTS credits |  |  | 5.0 |  |
| Learning profile | general academic profile |  | Assessment form |  |  | exam |  |
| Conducting unit | Mathematics Center -> Vice-Rector for Education |  |  |  |  |  |  |
| Name and surname of lecturer (lecturers) | Subject supervisor |  | dr inż. Magdalena Łapińska |  |  |  |  |
|  | Teachers |  | dr inż. Magdalena Łapińska |  |  |  |  |
| Lesson types and methods of instruction | Lesson type | Lecture | Tutorial | Laboratory | Project | Seminar | SUM |
|  | Number of study hours | 30.0 | 30.0 | 0.0 | 0.0 | 0.0 | 60 |
|  | E-learning hours included: 0.0 |  |  |  |  |  |  |
| Learning activity and number of study hours | Learning activity | Participation in didactic classes included in study plan |  | Participation in consultation hours |  | Self-study | SUM |
|  | Number of study hours | 60 |  | 12.0 |  | 53.0 | 125 |
| Subject objectives | The aim of the course is to give students a thorough understanding of basic concepts of calculus and algebra so that they are able to use them at different areas of economics. <br> After completing the course the student: <br> 1. will be provided with the ability of understanding the concepts of mathematical notions introduced during the lectures; <br> 2. will have developed competent skills and will be able to demonstrate problem solving skills at the areas of economics involving mathematical tools |  |  |  |  |  |  |


| Learning outcomes | Course outcome | Subject outcome | Method of verification |
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|  | [K6_W07] knows statistical and IT methods and tools that enable to obtain and present data on the organisation's resources | Student mentions basic properties of elementary functions. <br> Student solves equations and inequalities with elementary functions. <br> Student defines the basic concepts of differential calculus of one variable. <br> Student determines intervals of monotonicity of a given functions and its extrema. <br> Student analyses the properties of functions on the basis of an examination of its first and second derivatives. <br> Student geometrically interprets the results of an examination of a graph of a function using the concept of limit, continuity and derivatives of functions. <br> Student uses methods of mathematical description of phenomena in the economical processes. | [SW2] Assessment of knowledge contained in presentation [SW1] Assessment of factual knowledge |
|  | [K6_U15] can improve oneself through the systematic acquisition of knowledge and skills | Student recognizes the importance of self-expanding knowledge. <br> Student recognizes the importance of skillful use of basic mathematical apparatus in terms of study economics and finance. | [SU1] Assessment of task fulfilment |
|  | [K6_W08] has a basic knowledge of the methods and tools used to conduct research related to particular areas of business activity | Student combines knowledge of mathematics with knowledge from other fields. | [SW1] Assessment of factual knowledge |
| Subject contents | Matrices. Some types of matrices, equal matrices. Matrix addition, scalar multiplication, transpose of a matrix, matrix multiplication. Properties of matrix multiplication, examples. Determinants, properties of determinants. Invertible matrices, methods of obtaining the inverse of a square matrix. Systems of linear equations: Cramer's rule, method of matrix inversion. Rank of a matrix, row echelon form, elementary operations, fundamental theorem for systems of linear equations. Linear dependence and independence of rows and columns, method of Gaussian elimination. Rectangular coordinate system, vectors in $\mathrm{R}^{2}$, length of a vector, scalar product, the angle between vectors. Vectors in $R^{3}$, lines, planes. Vectors in $R^{n}$, lines in $R^{n}$, hyperplanes, flats. Linear and metric spaces, examples. Normed spaces, examples. Examples of applications in economics. Commodity bundle, the Leontief open production model. A simple example of application of linear programming in industry. <br> Basics of logic and set theory - Propositional calculus. Basic tautologies, statement forms, quantifiers. Sets and basic operations with sets, Cartesian products, relations, functions as relations. <br> Real valued functions of one variable. Functions and their properties: composite functions, inverse functions, inverses of the elementary functions. Infinite sequences, limit of a sequence, the limits laws. Computational techniques. Limit of a function, one-sided limits, properties of limits. Computational techniques. Continuous functions and their properties, points of discontinuity, examples. <br> Derivatives. Existence of derivatives, differentiation rules, the chain rule, derivatives of inverse functions. Calculation of derivatives of elementary functions and their inverses, derivatives of combinations of functions. Higher order derivatives. Taylor series for functions of one variable. Applications of derivatives . L'Hôpital's rule, Indeterminate forms. Asymptotes. Intervals of monotonicity, local and absolute extrema. |  |  |
| Prerequisites and co-requisites | No requirements |  |  |
| Assessment methods and criteria | Subject passing criteria | Passing threshold | Percentage of the final grade |
|  | Midterm colloquium | 50.0\% | 60.0\% |
|  | Final exam | 50.0\% | 35.0\% |
|  | e-Test | 50.0\% | 5.0\% |


| Recommended reading | Basic literature | 1. Martin Anthony, Norman Biggs, Mathematics for Economics and Finance Methods and Modelling, Cambridge University Press ISBN:0521559138. <br> 2. Hoffmann Laurence D., Bradley Gerald, Calculus for business, economics and the social and life sciences, <br> New York, McGraw-Hill Company, 1986, ISBN 978-0077292737 <br> 3. T. Jankowski, Linear Algebra, Wydawnictwo Politechniki Gdańskiej, Gdańsk 2001, ISBN 83-88007-87-4 |
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|  | Supplementary literature | No requirements |
|  | eResources addresses | Adresy na platformie eNauczanie: <br> WZiE - Z - Mathematics 2022/2023 (M.Łapińska) - Moodle ID: 24370 https://enauczanie.pg.edu.pl/moodle/course/view.php?id=24370 |
| Example issues/ example questions/ tasks being completed | 1. Suppose that an investor invests her money in three different assets and that three possible states can occur. Show that if the return matrix is R then Y and Z are arbitrage portfolios. Which of the two would you choose, given the choice? <br> 2. <br> The production processes for three goods $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}$ are interlinked. To produce one dollar's worth of $\mathrm{C}_{1}$ requires the input of $\$ 0.2$ worth of $\mathrm{C}_{1}, \$ 0.2$ of $\mathrm{C}_{2}$ and $\$ 0.1$ of $\mathrm{C}_{3}$. To produce one dollar's worth of $\mathrm{C}_{2}$ requires $\$ 0.1$ worth of $\mathrm{C}_{1}, \$ 0.2$ worth of $\mathrm{C}_{2}$ and $\$ 0.1$ worth of $\mathrm{C}_{3}$, and to produce one dollar's worth of $\mathrm{C}_{3}$ requires $\$ 0.1$ worth of each of $\mathrm{C}_{1}, \mathrm{C}_{2}$ and $\$ 0.2$ worth of $\mathrm{C}_{3}$. Suppose that in a given period, there is an external demand for 200 dollars' worth of $\mathrm{C}_{1}, 400$ of $\mathrm{C}_{2}$ and 300 of $\mathrm{C}_{3}$. We wish to know the production levels $x_{1}, x_{2}, x_{3}$ of $C_{1}, C_{2}, C_{3}$ required to satisfy all demands in the given period. <br> 3. A firm manufactures 3 different types of some good ' $A$ ', ' $B$ ' and ' $C$ '. The main ingredients in each are ' $a$ ', ' $b$ ' and ' 'c'. To produce 100 units of ' $A$ ' requires 1 units of ' $a$ ', 3 units of ' $b$ ' and 5 units of ' $c$ '. To produce 100 units of ' $B$ ' requires 4 units of ' $a$ ', 3 units of ' $b$ ' and 2 units of ' $c$ '. To produce 100 units of ' $C$ ' requires 2 units of 'a', 2 units of 'b' and 2 units of 'c'. The firm has supplies of 450 units of 'a', 360 of ' $b$ ' and 270 of 'c' each week (and as much as it wants of the other ingredients). How does the number of 'A' produced relate to the production level of the other two goods if the firm uses up its supply of 'a', 'b' and 'c'? Find the maximum possible weekly production of ' C '. <br> 4. <br> Find the time-independent solution of the recurrent equation $4 y t=y(t-1)+9, \quad(t=1,2,3, \ldots$. . Find the solution when $\mathrm{y} 0=6$, and describe its behaviour as $t$ tends to infinity. <br> 5. magine you have \$200 000 to invest, at a constant rate of $5 \%$, and that you want to withdraw a fixed at the end of each year for next twenty years. What is the maximum possible value of I for which this is possible? Answer the same question if withdrawals of I are to be made at the beginning of each of the next twenty years (including the present year). Find the local extrema of the given function $f(x)=x^{2} e^{-x}$ <br> 7. The function $g$ is given by $g(x)=x^{3}-6 x^{2}+12 x-1$. Show that $g$ has only one critical point. Determine whether this point is a maximum, a minimum, or an inflection point. Find asymptotes of the given function $y=x+2+1 /(x-2)$. <br> 9. Marginal cost function is defined to be the the derivative of the cost function. A manufacturer's cost function is $C(q)=1000+20 q+q(1+q)^{0.5}$. Find the marginal cost function. |  |
| Work placement | Not applicable |  |

