Subject card

| Subject name and code | Differential equations I, PG_00021499 |  |  |  |  |  |  |
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| Field of study | Mathematics |  |  |  |  |  |  |
| Date of commencement of studies | October 2023 |  | Academic year of realisation of subject |  |  | 2024/2025 |  |
| Education level | first-cycle studies |  | Subject group |  |  | Obligatory subject group in the field of study <br> Subject group related to scientific research in the field of study |  |
| Mode of study | Full-time studies |  | Mode of delivery |  |  | at the university |  |
| Year of study | 2 |  | Language of instruction |  |  | Polish |  |
| Semester of study | 3 |  | ECTS credits |  |  | 5.0 |  |
| Learning profile | general academic profile |  | Assessment form |  |  | exam |  |
| Conducting unit | Department of Nonlinear Analysis and Statistics -> Faculty of Applied Physics and Mathematics |  |  |  |  |  |  |
| Name and surname of lecturer (lecturers) | Subject supervisor |  | dr inż. Robert Krawczyk |  |  |  |  |
|  | Teachers |  | dr inż. Robert Krawczyk |  |  |  |  |
| Lesson types and methods of instruction | Lesson type | Lecture | Tutorial | Laboratory | Project | Seminar | SUM |
|  | Number of study hours | 30.0 | 30.0 | 0.0 | 0.0 | 0.0 | 60 |
|  | E-learning hours included: 0.0 |  |  |  |  |  |  |
|  | Adresy na platformie eNauczanie: |  |  |  |  |  |  |
| Learning activity and number of study hours | Learning activity | Participation in didactic classes included in study plan |  | Participation in consultation hours |  | Self-study | SUM |
|  | Number of study hours | 60 |  | 5.0 |  | 60.0 | 125 |
| Subject objectives | 1. solving the basic types of differential equations and systems of differential equations; <br> 2. investigating solutions of differential equations (existence and uniqueness of solution, extension, continuous dependence on initial conditions and parameters); <br> 3. description of simple phenomenas using differential equations. |  |  |  |  |  |  |


| Learning outcomes | Course outcome | Subject outcome | Method of verification |
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|  | K6_U09 | Student is able to define the domain of a solution of a differential equation depending on the initial condition. He/she knows the geometric interpretation of the solution to the ordinary differential equation. | [SU1] Assessment of task fulfilment [SU2] Assessment of ability to analyse information [SU4] Assessment of ability to use methods and tools |
|  | K6_W03 | Student is able to build a model of a differential equation describing a simple mathematical model used in geometry, economics and statistics. | [SW1] Assessment of factual knowledge |
|  | K6_U08 | The student is able to use all the basic concepts of linear algebra such as matrix, matrix determinant, eigenvalues and eigenvectors of matrices, the basis of a linear space. Kernel of linear mapping. The student uses these concepts to determine the fundamental matrix of a system of first-order ordinary differential equations, to determine the linear independence of the solutions of the fundamental system, to solve the system of differential equations with constant coefficients and the n-th order linear differential equation with constant coefficients. | [SU1] Assessment of task fulfilment [SU2] Assessment of ability to analyse information |
|  | K6_U01 | Student is able to formulate basic theorems from the theory of ordinary differential equations such as the theorem on the existence and uniqueness of a solution to a differential equation in the local and global version, theorems about the continuous dependence of solutions on parameters and initial conditions (Gronwall lemma). The student can use the Banach Fixed Point Theorem to solve simple firstorder linear differential equations. | [SU1] Assessment of task fulfilment [SU2] Assessment of ability to analyse information |
| Subject contents | 1. Applications leading to differential equations. The notions of a differential equation, its solution and an initial value problem. Geometric interpretation. Introductory remarks about existence and uniqueness of solution of an initial value problem. <br> 2. Separable differential equations. Existence and uniqueness of solution of separable equations. Methods of solution. <br> 3. Change of variables in differential equation. Linear and homogeneous equations. <br> 4. Differential equation of inverse function to the solution of differential equation. Bernoulli and Riccati differential equations. <br> 5. Exact differential equation. Integrating factor. Symmetrical form of differential equation of order one. <br> 6. Change of variables in differential equation of symmetrical form. Reduction of differential equation of order n to a system of differential equations of order one. Linear differential equations of order n . <br> 7. Factorization of linear differential operator of order $n$. Linear differential operators of order one. General solution of linear homogeneous equation of order $n$. <br> 8. Fundamental system of solutions. Constant coefficient nonhomogeneous linear equation of order n. <br> 9. Real solutions to constant coefficient nonhomogeneous linear equation of order n . Laplace method. <br> 10. A theorem about existence and uniqueness of solution to Cauchy problem. The Picard-Lindeloff theorem. The Peano theorem about the existence of solution to initial value problem. <br> 11. Continuous dependence of solution on initial conditions and parameters. Differentiability of solution with respect to initial conditions. <br> 12. Basic properties of solutions of linear systems of differential equations of order one (linear space of solutions to a homogeneous linear system of differential equations, its dimension and basis fundamental system, Wronski's matrix and the wronskian. <br> 13. The Liouville theorem. Solving linear nonhomogeneous systems using fundamental matrix of solutions of homogeneous systems. <br> 14. Solving constant coefficient linear homogeneous systems. Solving constant coefficient linear differential equations of higher order. <br> 15. Boundary problems for linear differential equations of order two. Sturm-Liouville boundary value problems. |  |  |
| Prerequisites and co-requisites | Calculus I and II, linear algebra |  |  |
| Assessment methods and criteria | Subject passing criteria | Passing threshold | Percentage of the final grade |
|  | Written form exam, exercises part | 50.0\% | 50.0\% |
|  | Written form exam, theoretical part | 50.0\% | 50.0\% |


| Recommended reading | Basic literature | 1. Z. Kamont, Równania różniczkowe zwyczajne, Wydawnictwo UG, Gdańsk, 1999. <br> 2. M. Kwapisz, Elementy zwyczajnych równań różniczkowych, Wydawnictwo UKW, Bydgoszcz, 2007. <br> 3. Muszyński, A.D Myszkis, Równania Różniczkowe Zwyczajne, PWN, Warszawa, 1984. <br> 4. A. Palczewski, Równania Różniczkowe Zwyczajne, WNT, Warszawa, 1999. <br> 5. A. Pelczar, J. Szarski, Wstęp do Teorii Równań Różniczkowych, cz. I,II, PWN, Warszawa, 1987, 1989. |
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|  | Supplementary literature | Trench W.F., Elementary Differential Equations, Free Edition 1.01 (December 2013) |
|  | eResources addresses |  |
| Example issues/ example questions/ tasks being completed | 1. Determine the region, where the Cauchy problem for the equation $y^{\prime}=1-\operatorname{ctg}(x)$ has a unique solution. <br> 2. Find the general solution to the differential equation $\left(x^{3}+e y\right) y^{\prime}=3 x^{2}$. <br> 3. Find the solution to the initial value problem $y^{\prime \prime \prime}-y^{\prime}=-2 x, y(0)=0, y^{\prime}(0)=1, y^{\prime \prime}(0)=2$. |  |
| Work placement | Not applicable |  |

