GDAŃSK UNIVERSITY OF TECHNOLOGY

Subject card

Subject name and code	Algebra I, PG_00023757						
Field of study	Mathematics						
Date of commencement of studies	October 2023		Academic year of realisation of subject	2024/2025			
Education level	first-cycle studies		Subject group	Obligatory subject group in the field of study Subject group related to scientific research in the field of study			
Mode of study	Full-time studies		Mode of delivery	at the university			
Year of study	2		Language of instruction	Polish			
Semester of study	4		ECTS credits	5.0			
Learning profile	general academic profile		Assessment form	exam			
Conducting unit	Zakład Analizy Nieliniowej -> Instytut Matematyki Stosowanej -> Faculty of Applied Physics and Mathematics						
Name and surname of lecturer (lecturers)	Subject supervisor		dr inż. Karol Wroński				
	Teachers						
Lesson types and methods of instruction	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	30.0	30.0	0.0	0.0	0.0	60
	E-learning hours included: 0.0						
	Adresy na platformie eNauczanie:						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours	Self-study	SUM	
	Number of study hours	60		5.0	60.0	125	
Subject objectives	Introduction to given aspects of the theory of groups, rings and fields.						

Learning outcomes

Course outcome	Subject outcome	Method of verification
K6_U08	The student learns the applications of algebra theorems in other areas of mathematics. In particular, these are properties of matrices and polynomials and the divisibility theory.	[SU2] Assessment of ability to analyse information [SU4] Assessment of ability to use methods and tools
K6_W04	The student knows the principle of mathematical induction and the properties of rational, real and complex integers important in algebra. He knows the properties of various types of matrices and operations on them. Applies facts from other subjects in algebra.	[SW1] Assessment of factual knowledge
K6_U01	The student recognizes and proves the properties of various groups and their subgroups referring to the correct statements. Finds generators in recursive groups. He applies the divisibility theory in the domains of integrity and Euclidean rings.	[SU2] Assessment of ability to analyse information [SU3] Assessment of ability to use knowledge gained from the subject
K6_U03	Student finds subgroups and normal divisors, constructs a quotient group. He gives examples of group homomorphisms and isomorphisms. Student finds sub-rings, ideals and constructs quotient rings. Creates extensions of the bodies.	[SU1] Assessment of task fulfilment [SU4] Assessment of ability to use methods and tools
K6_W02	The student shows the existence or non-existence of an isomorphism of various groups or rings. He proves the properties of quotient groups using appropriate isomorphism theorems.	[SW1] Assessment of factual knowledge

Subject contents

1. Group, subgroup, cosets, subgroup index, Lagrange theorem, examples.
2. Group homomorphism, normal subgroup, quotient group.
3. Isomorphism theorems, cyclic groups, structures of finitely generated abelian groups, group action, permutation groups.
4. Rings, zero divisors, invertible elements, ring homomorphism, ideals and examples.
5. Quotient ring - applications to the number theory (Fermat's Little Theorem, Z/m ring in diophantic equations).
6. Fields and prime fields, field isomorphism, subfields, field characteristic.
7. Polynomial rings, polynomial division, polynomial roots.
8. Roots of degree n.
9. Integral domain, field of fractions, kernels of homomorphisms onto integral domains - prime ideals, kernels of homomorphism onto fields - maximal ideals, existence of maximal ideals.
10. Irreducible elements of rings, unique factorisation domains.
11. Prime elements, greatest common divisor.
12. Principal ideal domains, euclidean rings, applications to equations over the integers.
13. Unique factorisation in polynomial rings, irreducible polynomials.
14. Algebraic field extension, algebraic elements and their properties, degree of an element and of an extension.
15. The field of algebraic elements. Splitting fields.
16. Algebraic closure. Fundamental theorem of algebra.

Prerequisites and co-requisites

Student should have passed the following courses:
- Linear algebra,
- Introduction to logic and set theory.

Assessment methods and criteria

Subject passing criteria	Passing threshold	Percentage of the final grade
Written exam	50.0%	40.0%
Tests during a semester	50.0%	60.0%

Recommended reading

Basic literature	1. Białynicki-Birula A. *Algebra*, PWN, Warszawa 1971. 2. Browkin J. *Teoria ciał*, PWN, Warszawa 1977. 3. Rutkowski J. Algebra abstrakcyjna w zadaniach, PWN, Warszawa 2004
Supplementary literature	1. Romanowski A., Turo J. *Algebra wyższa, zadania*, Wydawnictwo Politechniki Gdańskiej, Gdańsk 2007. 2. Lang S. *Algebra*, Springer Science+Business Media Inc., 2002. 3. Kostrykin A., Wstęp do Algebry, t. I-III, PWN 2009
eResources addresses	

Example issues/ example questions/ tasks being completed	1. Check if the set of invertible matrices with rational determinants is a subgroup of the group of real invertible matrices. 2. Determine conjugacy classes in the group S_7. 3. Show that **R** i $\mathbf{R}^*$ are not isomorphic groups. 4. Show that a function f:$\mathbf{C}^*\mathbf{C}^*$,defined by f(z)=|z|, is a group homomorphism. Determine its kernel and image. 5. Check if a given set is a subring of a given ring R. 6. Check if a given set is an ideal of a given ring R. 7. Determine associates of the element 12-i in the ring **Z**[i]. 8. Determine quotient and reminder of division $2x^5+5x^4+x^3+2x^2+3x+5$ by x+3 in the ring $\mathbf{Z}_7[x]$. 9. Determine the multiplicity of the root $x_0=-2$ of the polynomial $x^5+7x^4+16x^3+8x^2-16x-16$ in the ring **Q**[x]. 10. Determine the degree of the algebraic number 1+i3.
Work placement	Not applicable