



Subject card

Subject name and code	Fraktals, PG_00021049						
Field of study	Mathematics						
Date of commencement of studies	October 2023	Academic year of realisation of subject			2023/2024		
Education level	second-cycle studies	Subject group			Optional subject group Subject group related to scientific research in the field of study		
Mode of study	Full-time studies	Mode of delivery			at the university		
Year of study	1	Language of instruction			Polish		
Semester of study	1	ECTS credits			4.0		
Learning profile	general academic profile	Assessment form			assessment		
Conducting unit	Zakład Układów Dynamicznych -> Instytut Matematyki Stosowanej -> Faculty of Applied Physics and Mathematics						
Name and surname of lecturer (lecturers)	Subject supervisor		prof. dr hab. Joanna Janczewska				
	Teachers		prof. dr hab. Joanna Janczewska				
Lesson types and methods of instruction	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	30.0	15.0	0.0	15.0	0.0	60
E-learning hours included: 0.0							
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours		Self-study	SUM
	Number of study hours	60		5.0		60.0	125
Subject objectives	The aim of the lecture is to introduce the central ideas and concepts of fractals as well as many related topics.						
Learning outcomes	Course outcome		Subject outcome		Method of verification		
	[K7_W05] Has enhanced knowledge of a selected branch of mathematics: knows most classical definitions and theorems and their proofs, Understands problems being examined, Knows relations between problems from particular field with other branches of mathematics, theoretical and applied		A student knows basic notions and facts of discreet and continuous dynamical systems. A student can define the space of Hausdorff type. A student is able to explain the notion of fractal by the use of examples from biology. A student can compute fractal dimension, Hausdorff's dimension and topological dimension.		[SW1] Assessment of factual knowledge		
	[K7_U09] Is able, at an advanced level and covering modern mathematics, to apply and present in speech and in writing the methods of at least one selected branch of mathematics: mathematical and functional analysis, theory of differential equations and dynamical systems, algebra and number theory, geometry and topology, calculus probability and statistics, discrete mathematics and graph theory, logic and set theory.		A student can apply the knowledge acquired at other lectures.		[SU3] Assessment of ability to use knowledge gained from the subject		
	[K7_U11] Can construct mathematical models used in specific advanced applications of mathematics, can use stochastic processes as a tool for modeling phenomena and analyzing their evolution.		A student understands relations between the fractal theory and dynamical systems.		[SU3] Assessment of ability to use knowledge gained from the subject		

Subject contents	The Banach contraction principle. Examples of fractals. Why do so many people study fractals? Fractal spaces with the Hausdorff metric. Iterated function systems (IFS). A fractal dimension, the Hausdorff dimension and a topological dimension. The Mandelbrot definition of fractals. Julia sets. The Mandelbrot set. Discrete dynamical systems. Continuous dynamical systems. A definition and properties of the Poincare map. Attractors and repellers. The Feigenbaum cascade. The Smale horseshoe - a geometric description. Properties of the invariant set of the Smale horseshoe.		
Prerequisites and co-requisites	Mathematical analysis. Topology. Ordinary differential equations.		
Assessment methods and criteria	Subject passing criteria	Passing threshold	Percentage of the final grade
	Project	100.0%	50.0%
	A maths test	50.0%	50.0%
Recommended reading	Basic literature	1. Jacek Kudrewicz, <i>Fraktale i Chaos</i> , Wydawnictwa Naukowo-Techniczne, Warszawa, 2007. 2. Lawrence Perko, <i>Differential Equations and Dynamical Systems</i> , Springer, New York, 2001.	
	Supplementary literature	1. J.D. Murray, <i>Mathematical Biology. I: An Introduction</i> , Springer-Verlag, New York, 2002. 2. H.-O. Peitgen, H. Jurgens, D. Saupe, <i>Chaos and Fractals. New Frontiers of Science</i> , Springer-Verlag, New York, 2004.	
	eResources addresses	Adresy na platformie eNauczenie: Fraktale rok akad. 2023/2024 - Moodle ID: 34585 https://enauczenie.pg.edu.pl/moodle/course/view.php?id=34585	
Example issues/ example questions/ tasks being completed	1. Is a given subset Z in \mathbb{R}^n compact (connected, nowhere dense)? Justify the answer. 2. Calculate the Hausdorff distance between two given subsets A and B in \mathbb{R}^2 . 3. Calculate a fractal dimension, the Hausdorff dimension and a topological dimension of the Cantor set, the Koch curve, the Sierpiński gasket and carpet. 4. Let $w_1, w_2, \dots, w_k: \mathbb{R}^n \rightarrow \mathbb{R}^n$ be given. Prove that $\{\mathbb{R}^n; w_1, w_2, \dots, w_k\}$ is an iterated function system. Calculate the constant of its contraction. 5. Solve a linear differential equation of first order $x' = Ax$ in \mathbb{R}^2 , where A is a given square matrix 2×2 . 6. Give a geometric description of the Smale horseshoe map. 7. What is it an attractor? Give a short description of the Hénon attractor, the Rössler attractor and the Lorenz attractor.		
Work placement	Not applicable		