

## Subject card

| Subject name and code  | Linear Algebra, PG_00021032  |  |   |                                     |        |  |                |           |  |
|--|--|--|---|-------------------------------------|--------|--|----------------|-----------|--|
| Field of study   | Mathematics  |  |   |                                     |        |  |                |           |  |
| Date of commencement of studies  | October 2024   |  | Academic year of realisation of subject |                                     |        | 2024/2025  |                |           |  |
| Education level  | first-cycle studies  |  | Subject group                           |                                     |        | Obligatory subject group in the field of study Subject group related to scientific |                |           |  |
|  |  |  |   |                                     |        | research in the field of study   |                |           |  |
| Mode of study  | Full-time studies  |  | Mode of delivery                        |                                     |        | at the university  |                |           |  |
| Year of study  | 1  |  | Language of instruction                 |                                     |        | Polish   |                |           |  |
| Semester of study  | 2  |  | ECTS credits                            |                                     |        | 5.0  |                |           |  |
| Learning profile   | general academic profile   |  | Assessment form                         |                                     | exam   |  |                |           |  |
| Conducting unit  | Department of Nonlinear Analysis and Statistics -> Faculty of Applied Physics and Mathematics  |  |   |                                     |        |  |                |           |  |
| Name and surname   | Subject supervisor   |  | dr Joanna Cyman                         |                                     |        |  |                |           |  |
| of lecturer (lecturers)  | Teachers   |  | dr Maryna Shcholokova                   |                                     |        |  |                |           |  |
|  | dr Joanna Cyman  |  |   |                                     |        |  |                |           |  |
| Lesson types and methods of instruction  | Lesson type  | Lecture  | Tutorial                                | Laboratory                          | Projec | t  | Seminar        | SUM       |  |
|  | Number of study hours  | 30.0   | 30.0                                    | 0.0                                 | 0.0    |  | 0.0            | 60        |  |
|  | E-learning hours included: 0.0   |  |   |                                     |        |  |                |           |  |
|  | Adresy na platformie eNauczanie:   |  |   |                                     |        |  |                |           |  |
|  | Algebra liniowa II 2024/2025 - Moodle ID: 42731 https://enauczanie.pg.edu.pl/moodle/course/view.php? id=42731                        |  |   |                                     |        |  |                |           |  |
| Learning activity and number of study hours  | Learning activity  | Participation in didactic classes included in study plan |   | Participation in consultation hours |        | Self-study   |                | SUM       |  |
|  | Number of study hours  | 60   |   | 5.0                                 |        | 60.0   |                | 125       |  |
| Subject objectives Introducing students to key concepts of vector spaces and linear fundamental knowledge of Euclidean spaces. |  |  |   |                                     |        | nations  | , as well as r | mastering |  |
|  | Developing the ability to apply the acquired knowledge to solve theoretical and practical problems in various fields of mathematics. |  |   |                                     |        |  |                |           |  |

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| Learning outcomes               | Course outcome  | Subject outcome  | Method of verification   |  |  |  |  |
|---------------------------------|---|--|--|--|--|--|--|
|                                 | K6_U01  | The student is able to prove simple algebraic properties and examine linear independence and orthogonality of vectors.   | [SU4] Assessment of ability to<br>use methods and tools<br>[SU2] Assessment of ability to<br>analyse information |  |  |  |  |
|                                 | K6_U08  | The student understands the concept of a vector space, is able to check the linear independence of a set of vectors, and find bases of subspaces. They can also find the coordinates of a vector in a given basis. The student understands the concept of a linear transformation and knows examples of such transformations. They can find the kernel and image of a linear transformation and understand the relationship between the dimension of the domain of the transformation and the dimensions of its kernel and image. They can find the matrix of a linear transformation in given bases. The student is able to calculate eigenvalues and eigenvectors. The student understands the concept of the scalar product and orthonormal bases. They can perform the Gram-Schmidt orthogonalization. Furthermore, they can analyze bilinear forms. | [SU4] Assessment of ability to use methods and tools   |  |  |  |  |
|                                 | K6_U03  | The student correctly defines various algebraic structures.  | [SU3] Assessment of ability to use knowledge gained from the subject   |  |  |  |  |
|                                 | K6_W02  | The student conducts simple reasoning, correctly formulates and proves theorems from linear algebra, such as the theorem on the basis of a space and the Rank–nullity theorem.   | [SW1] Assessment of factual knowledge  |  |  |  |  |
|                                 | K6_W07  | The student acquires the skills to apply the concepts of linear algebra in various fields of mathematics.  | [SW1] Assessment of factual knowledge  |  |  |  |  |
| Subject contents                | Vector space. Basis and dimension of vector space. Coordinates of a vector in the vector space basis. T change-of-basis matrix.  Linear maps. Kernel and image. Matrix of a linear map. Operations on maps. |  |  |  |  |  |  |
|                                 | Euclidean spaces. Scalar product, orthogonality of vectors, orthogonal and orthonormal basis.  GramSchmidt process.   |  |  |  |  |  |  |
|                                 | <b>Eigenvalues and eigenvectors.</b> Eigenvalues and eigenvectors of matrices and mappings. Cayley-Hamilton theorem.  |  |  |  |  |  |  |
|                                 | Quadratic form. Real quadratic form. Quadratic form in canonical form.  |  |  |  |  |  |  |
| Prerequisites and co-requisites | Linear Algebra I  |  |  |  |  |  |  |
| Assessment methods              | Subject passing criteria  | Passing threshold  | Percentage of the final grade  |  |  |  |  |
| and criteria                    | colloquia   | 50.0%  | 50.0%  |  |  |  |  |
|                                 | exam  | 50.0%  | 40.0%  |  |  |  |  |
|                                 | activity  | 0.0%   | 10.0%  |  |  |  |  |

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| Recommended reading                | Basic literature   | T. Jurlewicz, Z. Skoczylas, Linear Algebra 1 and 2. Definitions, theorems, formulas, Oficyna Wydawnicza GiS, Wrocław 2023/2024.  |  |  |  |
|------------------------------------|--|--|--|--|--|
|                                    |  | T. Jurlewicz, Z. Skoczylas, Linear Algebra 1 and 2. Examples and tasks, Oficyna Wydawnicza GiS, Wrocław 2023/2024.   |  |  |  |
|                                    |  | J. Topp, Linear algebra, University of Gdańsk Publishing House,<br>Gdańsk 2015.  |  |  |  |
|                                    | Supplementary literature   | J. Rutkowski, Linear algebra in tasks, PWN 2008  |  |  |  |
|                                    |  | G. Banaszak, W. Gajda, Elements of linear algebra, WNT 2002.   |  |  |  |
|                                    |  | A. I. Kostrikin, Introduction to Algebra. PWN, Warszawa 2024.  |  |  |  |
|                                    | eResources addresses   | Podstawowe https://enauczanie.pg.edu.pl/moodle/course/view.php?id=42731 - Algebra liniowa II 2024/2025 - Moodle ID: 42731 https://enauczanie.pg.edu.pl/moodle/course/view.php?id=42731 |  |  |  |
| Example issues/ example questions/ | 1. Given a matrix A:   | https://citadozanie.pg.cad.pi/nioodic/codice/view.php.id=42701   |  |  |  |
| tasks being completed              | a) Find its eigenvalues and eigenvectors.  |  |  |  |  |
|                                    | b) Is A diagonalizable? Justify your answer. c) Using the Cayley-Hamilton theorem, compute A^(-1).   |  |  |  |  |
|                                    |  |  |  |  |  |
|                                    |  |  |  |  |  |
|                                    | <ul><li>2. Check which of the following sets is a subspace of the vector space R^2:</li><li>a) A = {(x, y) R^2 :  x  =  y };</li></ul>   |  |  |  |  |
|                                    |  |  |  |  |  |
|                                    | b) B = {(x, y) R^2 : x = 3y};<br>c) C = {(x, y) R^2 : x = y + 3}.  |  |  |  |  |
|                                    |  |  |  |  |  |
|                                    |  |  |  |  |  |
|                                    | <ul> <li>3. Given the linear transformation T: R^4 R^3, T(x, y, z, t) = (x + y - z, -x - y + z, x + 3t):</li> <li>a) Find the kernel, the image, and their bases;</li> <li>b) Orthogonalize these bases using the Gram-Schmidt method;</li> <li>c) Find the coordinates of the vectors (5, -5, 6) and (6, -2, 4, -2) in the chosen bases.</li> </ul> |  |  |  |  |
|                                    |  |  |  |  |  |
|                                    |  |  |  |  |  |
|                                    |  |  |  |  |  |
|                                    | 4. State and prove the Ranknullity theorem.  |  |  |  |  |
| Work placement                     | Not applicable   |  |  |  |  |

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