



Subject card

Subject name and code	Fraktals, PG_00021049						
Field of study	Mathematics						
Date of commencement of studies	October 2024	Academic year of realisation of subject			2024/2025		
Education level	second-cycle studies	Subject group			Specialty subject group Subject group related to scientific research in the field of study		
Mode of study	Full-time studies	Mode of delivery			at the university		
Year of study	1	Language of instruction			Polish		
Semester of study	1	ECTS credits			4.0		
Learning profile	general academic profile	Assessment form			assessment		
Conducting unit	Zakład Układów Dynamicznych -> Instytut Matematyki Stosowanej -> Faculty of Applied Physics and Mathematics						
Name and surname of lecturer (lecturers)	Subject supervisor	prof. dr hab. Joanna Janczewska					
	Teachers	prof. dr hab. Joanna Janczewska					
Lesson types and methods of instruction	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	30.0	15.0	0.0	15.0	0.0	60
	E-learning hours included: 0.0						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan	Participation in consultation hours		Self-study		SUM
	Number of study hours	60	5.0		60.0		125
Subject objectives	The aim of the lecture is to introduce the central ideas and concepts of fractals.						
Learning outcomes	Course outcome	Subject outcome			Method of verification		
	[K7_W02] has enhanced knowledge of a selected branch of mathematics, theoretical or applied, knows classical definitions and theorems and their proofs and connections with other fields, understands problems being examined	A student has in-depth knowledge of fractal geometry.			[SW3] Assessment of knowledge contained in written work and projects		
	[K7_U07] at an advanced level and covering modern mathematics, applies and presents in speech and in writing the content and methods of a selected branch of mathematics	A student applies and presents selected content and methods of fractal geometry.			[SU3] Assessment of ability to use knowledge gained from the subject		
	[K7_U09] constructs mathematical models used in specific advanced applications of mathematics, can use stochastic processes as a tool for modeling phenomena and analyzing their evolution, constructs mathematical models used in specific advanced applications of mathematics, uses stochastic processes as a tool for modeling phenomena and analyzing their evolution, recognizes mathematical structures in physical theories	A student applies selected notions and methods from dynamical systems in fractal geometry.			[SU3] Assessment of ability to use knowledge gained from the subject		
	[K7_U03] uses differential and integral calculus, elements of complex analysis, algebraic methods, applies them in typical practical	A student uses knowledge of analysis, topology, algebra and geometry in fractal geometry.			[SU4] Assessment of ability to use methods and tools		

Subject contents	The Banach contraction principle. Examples of fractals. Why do so many people study fractals? Fractal spaces with the Hausdorff metric. Iterated function systems (IFS). A fractal dimension, the Hausdorff dimension and a topological dimension. The Mandelbrot definition of fractals. Julia sets. The Mandelbrot set. Discrete dynamical systems. Continuous dynamical systems. A definition and properties of the Poincaré map. Attractors and repellers. The Feigenbaum cascade. The Smale horseshoe - a geometric description. Properties of the invariant set of the Smale horseshoe.		
Prerequisites and co-requisites	Mathematical analysis. Topology. Ordinary differential equations.		
Assessment methods and criteria	Subject passing criteria	Passing threshold	Percentage of the final grade
	A maths test	50.0%	50.0%
	Project	100.0%	50.0%
Recommended reading	Basic literature	1. Jacek Kudrewicz, <i>Fraktale i Chaos</i> , Wydawnictwa Naukowo-Techniczne, Warszawa, 2007. 2. Lawrence Perko, <i>Differential Equations and Dynamical Systems</i> , Springer, New York, 2001.	
	Supplementary literature	1. J.D. Murray, <i>Mathematical Biology. I: An Introduction</i> , Springer-Verlag, New York, 2002. 2. H.-O. Peitgen, H. Jurgens, D. Saupe, <i>Chaos and Fractals. New Frontiers of Science</i> , Springer-Verlag, New York, 2004.	
	eResources addresses	Adresy na platformie eNauczanie:	
Example issues/ example questions/ tasks being completed	1. Is a given subset Z in \mathbb{R}^n compact (connected, nowhere dense)? Justify the answer. 2. Calculate the Hausdorff distance between two given subsets A and B in \mathbb{R}^2 . 3. Calculate a fractal dimension, the Hausdorff dimension and a topological dimension of the Cantor set, the Koch curve, the Sierpiński gasket and carpet. 4. Let $w_1, w_2, \dots, w_k: \mathbb{R}^n \rightarrow \mathbb{R}^n$ be given. Prove that $\{\mathbb{R}^n; w_1, w_2, \dots, w_k\}$ is an iterated function system. Calculate the constant of its contraction. 5. Solve a linear differential equation of first order $x' = Ax$ in \mathbb{R}^2 , where A is a given square matrix 2×2 . 6. Give a geometric description of the Smale horseshoe map. 7. What is it an attractor? Give a short description of the Hénon attractor, the Rössler attractor and the Lorenz attractor.		
Work placement	Not applicable		

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