GDAŃSK UNIVERSITY OF TECHNOLOGY

Subject card

Subject name and code	Introduction to the Calculus of Variation, PG_00069088		
Field of study	Technical Physics, Materials Engineering, Mathematics, Nanotechnology, Nanotechnology		
Date of commencement of studies	October 2024	Academic year of realisation of subject	2025/2026
Education level	second-cycle studies	Subject group	Optional subject group
Mode of study	Full-time studies	Mode of delivery	at the university
Year of study	2	Language of instruction	Polish
Semester of study	3	ECTS credits	1.0
Learning profile	general academic profile	Assessment form	assessment
Conducting unit	Institute of Applied Mathematics -> Faculty of Applied Physics and Mathematics -> Wydziały Politechniki Gdańskiej		
Name and surname of lecturer (lecturers)	Subject supervisor	dr hab. Sergey Kryzhevich	
	Teachers	dr hab. Sergey Kryzhevich	

Lesson types and methods of instruction

Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
Number of study hours	15.0	0.0	0.0	0.0	0.0	15

E-learning hours included: 0.0

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Learning activity and number of study hours

Learning activity	Participation in didactic classes included in study plan	Participation in consultation hours	Self-study	SUM
Number of study hours	15	2.0	8.0	25

Subject objectives

The objective of the course is the introduction to the the basic methods of the Calculus of Variations and development of mathematical models in the form of ordinary differential equations.

Learning outcomes

Course outcome	Subject outcome	Method of verification
[K7_K04] forms opinions on mathematical issues	A student is able to construct the integral functional that corresponds to the given variational problem, to derive the Euler-Lagrange equation and to solve it with the appropriate boundary conditions.	[SK5] Assessment of ability to solve problems that arise in practice
[K7_W02] has enhanced knowledge of a selected branch of mathematics,theoretical or applied, knows classical definitions and theorems and their proofs and connections with other fields,understands problems being examined	A student is able to formulate and prove the basic statements of the Variational Calculus Theory and understands the related terminology. They understand how the differential equations arise from physical principles.	[SW1] Assessment of factual knowledge

Subject contents	1. Fundamental Lemma of the Calculus of Variations. 2. The Quadratic Variational Problem. Optimality Criterion. 3. Legendre's Necessary Condition for the Non-Negative Definiteness of a Quadratic Integral Form. The Criterion of Non-Negative Definiteness of a Quadratic Integral Form. Proof of Sufficiency. 4. The Corner Rounding Lemma. The Criterion of Non-Negative Definiteness of a Quadratic Integral Form. Proof of Necessity. 5. The Criterion of Positive Definiteness of a Quadratic Integral Form. Lower Boundary of a Positively Definite Form of a Quadratic Integral. 6. Description of the Entire Solution Set of the Quadratic Variational Problem. Solution Scheme for the Quadratic Variational Problem. Example. 7. The Nonlinear Variational Problem. Finite-Dimensional Approximation. The Concept of the Variational Derivative. 8. The Natural Domain of the Definition of an Integral Functional. Its openness in the space of continuously differentiable functions. 9. The first and second derivatives of an integral functional. 10. Necessary conditions for a first-order local minimum in a nonlinear variational problem (in terms of the first differential and the original problem). Theorem on the existence and continuity of the second derivative of the extremum of a nonlinear variational problem. 11. Necessary conditions for a second-order local minimum in a nonlinear variational problem (in terms of the second differential and the original problem). Sufficient conditions for a strict local minimum in a nonlinear variational problem. 12. A parametric method for constructing a solution to Jacobi's principal equation. Example. 13. The minimum surface of revolution problem. 14. The case of two stationary curves in the minimum surface of revolution problem. 15. The isoperimetric problem. A chain curve.
Prerequisites and co-requisites	The knowledge of the basic concepts and statements of Mathematical Analysis and Ordinary Differential Equations is required.

Assessment methods and criteria

Subject passing criteria	Passing threshold	Percentage of the final grade
Colloquium	51.0%	100.0%

Recommended reading

Basic literature	1. Jost, J. and X. Li-Jost: Calculus of Variations. Cambridge University Press, 1998. 2. Gelfand, I. M.; Fomin, S. V. (2000). Silverman, Richard A. (ed.). Calculus of variations (Unabridged repr. ed.). Mineola, New York: Dover Publications. p. 3. ISBN 978-0486414485.
Supplementary literature	R. Weinstock, Calculus of Variations with Applications to Physics and Engineering, Dover, 1974 (reprint of 1952 ed.).
eResources addresses	Basic https://enauczanie.pg.edu.pl/2025/course/view.php?id=1124 - Address at the eNauczanie platform introduction to the variational calculus 2025/2026 ID 1124

Example issues/ example questions/ tasks being completed	1. Deriving Friedrichs' inequality for functions of one variable with n=2. 2. Solving the brachistochrone problem. 3. Equations of straight lines in Lobachevsky geometry.
Work placement	Not applicable

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