



Subject card

Subject name and code	, PG_00066266						
Field of study	Mathematics						
Date of commencement of studies	October 2025		Academic year of realisation of subject		2025/2026		
Education level	second-cycle studies		Subject group		Specialty subject group Subject group related to scientific research in the field of study		
Mode of study	Full-time studies		Mode of delivery		at the university		
Year of study	1		Language of instruction		Polish		
Semester of study	1		ECTS credits		4.0		
Learning profile	general academic profile		Assessment form		assessment		
Conducting unit	Division of Dynamical Systems -> Institute of Applied Mathematics -> Faculty of Applied Physics and Mathematics -> Wydziały Politechniki Gdańskiej						
Name and surname of lecturer (lecturers)	Subject supervisor		prof. dr hab. Joanna Janczewska				
	Teachers		prof. dr hab. Joanna Janczewska				
Lesson types and methods of instruction	Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
	Number of study hours	30.0	15.0	0.0	15.0	0.0	60
	E-learning hours included: 0.0						
	eNauczanie source addresses: Moodle ID: 1307 Fraktale 2025/2026 https://enauczanie.pg.edu.pl/2025/course/view.php?id=1307						
Learning activity and number of study hours	Learning activity	Participation in didactic classes included in study plan		Participation in consultation hours		Self-study	SUM
	Number of study hours	60		5.0		35.0	100
Subject objectives	The aim of the lecture is to introduce the central ideas and concepts of fractals.						

Learning outcomes	Course outcome	Subject outcome	Method of verification
	[K7_U03] uses differential and integral calculus, elements of complex analysis, algebraic methods, applies them in typical practical	A student uses knowledge of analysis, topology, algebra and geometry in fractal geometry.	[SU4] Assessment of ability to use methods and tools
	[K7_U09] constructs mathematical models used in specific advanced applications of mathematics, can use stochastic processes as a tool for modeling phenomena and analyzing their evolution, constructs mathematical models used in specific advanced applications of mathematics, uses stochastic processes as a tool for modeling phenomena and analyzing their evolution, recognizes mathematical structures in physical theories	A student applies selected notions and methods from dynamical systems in fractal geometry.	[SU3] Assessment of ability to use knowledge gained from the subject
	[K7_U07] at an advanced level and covering modern mathematics, applies and presents in speech and in writing the content and methods of a selected branch of mathematics	A student applies and presents selected content and methods of fractal geometry.	[SU3] Assessment of ability to use knowledge gained from the subject
	[K7_W02] has enhanced knowledge of a selected branch of mathematics, theoretical or applied, knows classical definitions and theorems and their proofs and connections with other fields, understands problems being examined	A student has in-depth knowledge of fractal geometry.	[SW3] Assessment of knowledge contained in written work and projects
Subject contents	The Banach contraction principle. Examples of fractals. Why do so many people study fractals? Fractal spaces with the Hausdorff metric. Iterated function systems (IFS). A fractal dimension, the Hausdorff dimension and a topological dimension. The Mandelbrot definition of fractals. Julia sets. The Mandelbrot set. Discrete dynamical systems. Continuous dynamical systems. A definition and properties of the Poincare map. Attractors and repellers. The Feigenbaum cascade. The Smale horseshoe - a geometric description. Properties of the invariant set of the Smale horseshoe.		
Prerequisites and co-requisites	Mathematical analysis. Topology. Ordinary differential equations.		
Assessment methods and criteria	Subject passing criteria	Passing threshold	Percentage of the final grade
	A maths test	50.0%	70.0%
	Project	50.0%	30.0%
Recommended reading	Basic literature	1. Jacek Kudrewicz, Fraktale i Chaos, Wydawnictwo Naukowe PWN SA, Warszawa, 2007. 2. Lawrence Perko, Differential Equations and Dynamical Systems, Springer, New York, 2001. 3. Andrzej Katunin, Fraktale. Matematyczne potwory, które odmieniły postrzeganie świata, Wydawnictwo PŚ, Gliwice, 2021.	
	Supplementary literature	1. J.D. Murray, Mathematical Biology. I: An Introduction, Springer-Verlag, New York, 2002. 2. H.-O. Peitgen, H. Jurgens, D. Saupe, Chaos and Fractals. New Frontiers of Science, Springer-Verlag, New York, 2004.	
	eResources addresses		

Example issues/ example questions/ tasks being completed	<p>1. Is a given subset Z in R^n compact (connected, nowhere dense)? Justify the answer.</p> <p>2. Calculate the Hausdorff distance between two given subsets A and B in R^2.</p> <p>3. Calculate a fractal dimension, the Hausdorff dimension and a topological dimension of the Cantor set, the Koch curve, the Sierpiński gasket and carpet.</p> <p>4. Let w_1, w_2, \dots, w_k from R^n to R^n be given. Prove that $\{R^n; w_1, w_2, \dots, w_k\}$ is an iterated function system. Calculate the constant of its contraction.</p> <p>5. Solve a linear differential equation of first order $x'=Ax$ in R^2, where A is a given square matrix 2×2.</p> <p>6. Give a geometric description of the Smale horseshoe map.</p> <p>7. What is it an attractor? Give a short description of the Hénon attractor, the Rössler attractor and the Lorenz attractor.</p>
Work placement	Not applicable

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