GDAŃSK UNIVERSITY OF TECHNOLOGY

Subject card

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|---|---|---|---|
| Subject name and code | , PG_00070313 | | |
| Field of study | Mathematics | | |
| Date of commencement of studies | October 2024 | Academic year of realisation of subject | 2025/2026 |
| Education level | second-cycle studies | Subject group | Specialty subject group<br>Subject group related to scientific research in the field of study |
| Mode of study | Full-time studies | Mode of delivery | at the university |
| Year of study | 2 | Language of instruction | Polish |
| Semester of study | 4 | ECTS credits | 2.0 |
| Learning profile | general academic profile | Assessment form | assessment |
| Conducting unit | Divison of Dynamical Systems -> Institute of Applied Mathematics -> Faculty of Applied Physics and Mathematics -> Faculties of Gdańsk University of Technology | | |
| Name and surname of lecturer (lecturers) | Subject supervisor | dr Klaudiusz Czudek | |
| | Teachers | dr Klaudiusz Czudek | |

**Lesson types**

| Lesson type | Lecture | Tutorial | Laboratory | Project | Seminar | SUM |
|---|---|---|---|---|---|---|
| Number of study hours | 30.0 | 0.0 | 0.0 | 0.0 | 30.0 | 60 |

E-learning hours included: 0.0

eNauczanie source address: https://enauczanie.pg.edu.pl/2025/course/view.php?id=5436

Moodle ID: 5436 Teoria ergodyczna
https://enauczanie.pg.edu.pl/2025/course/view.php?id=5436

**Learning activity and number of study hours**

| Learning activity | Participation in didactic classes included in study plan | Participation in consultation hours | Self-study | SUM |
|---|---|---|---|---|
| Number of study hours | 60 | 0.0 | 0.0 | 60 |

**Subject objectives**

In this course I am going to introduce basic notions of ergodic theory: measure preserving systems, Bikhoff theorem, mixing systems, weak mixing systems, Bernoulli and Kolmogorov systems. We will investigate basic examples of systems with those properties and origins of ergodic theory in physics. Spectral theory of Koopman operator and metric entropy are going to be discussed.

**Learning outcomes**

| Course outcome | Subject outcome | Method of verification |
|---|---|---|
| [K7_W03] demonstrates knowledge advanced computation techniques, supporting the work of a mathematician and understand their limitations. | A student understands how to exploit a computer to simulate dynamical systems | [SW1] Assessment of factual knowledge |
| [K7_U10] understands the mathematical foundations of the analysis of algorithms and computational processes, constructs algorithms with good numerical properties, used to solve typical and unusual mathematical problems | A student is able to estimate the complexity of algorithms used to study the trajectories of dynamical systems | [SU3] Assessment of ability to use knowledge gained from the subject<br>[SU4] Assessment of ability to use methods and tools |
| [K7_W02] has enhanced knowledge of a selected branch of mathematics,theoretical or applied, knows classical definitions and theorems and their proofs and connections with other fields,understands problems being examined | A student gives basic definitions of ergodic theory | [SW1] Assessment of factual knowledge |

<table>
<tr><td>Subject contents</td><td>Course content – lecture<br>1. Hamiltonians, measure preserving systems, Liouville's theorem.<br><br>2. Ideal gas, Clapeyron equation, equipartition of energy, Boltzmann ergodic hypothesis.<br><br>3. Geodesic flow on surfaces with negative curvature. Hard spheres system.<br><br>4. Kac' lemma, Poincare recurrence theorem.<br><br>5. Differential equations on torus. Ergodic properties of the circle rotation.<br><br>6.Ergodic theory of Markov processes, Ito diffusion, infinitesimal operators.<br><br>7. Spectral theory of Koopman operators, entropy.<br><br>Course content – seminar<br>Solving problems related to the content of lectures.</td></tr>
<tr><td>Prerequisites and co-requisites</td><td>Probability theory, stochastic integral, stochastic differential equations</td></tr>
<tr><td>Assessment methods and criteria</td><td><table>
<tr><th>Subject passing criteria</th><th>Passing threshold</th><th>Percentage of the final grade</th></tr>
<tr><td>Test</td><td>50.0%</td><td>100.0%</td></tr>
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<tr><td>Recommended reading</td><td><table>
<tr><td>Basic literature</td><td>Cornfeld, Fomin, Sinai "Ergodic theory"</td></tr>
<tr><td>Supplementary literature</td><td>Katok, Hasselblatt "Introduction to the modern theory of dynamical systems" 1995<br><br>Kallenberg "Foundations of modern probability" 2002</td></tr>
<tr><td>eResources addresses</td><td></td></tr>
</table></td></tr>
<tr><td>Example issues/ example questions/ tasks being completed</td><td>Prove that a given system preserves a given measure. Prove ergodicity/weak mixing/mixing of the system.</td></tr>
<tr><td>Practical activites within the subject</td><td>Not applicable</td></tr>
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