GDAŃSK UNIVERSITY OF TECHNOLOGY

Subject card

Subject name and code	Introduction to the Ergodic Theory, PG_00071151		
Field of study	Technical Physics, Mathematics, Nanotechnology, Nanotechnology		
Date of commencement of studies	October 2024	Academic year of realisation of subject	2025/2026
Education level	second-cycle studies	Subject group	Optional subject group
Mode of study	Full-time studies	Mode of delivery	at the university
Year of study	2	Language of instruction	Polish
Semester of study	4	ECTS credits	1.0
Learning profile	general academic profile	Assessment form	assessment
Conducting unit	Institute of Applied Mathematics -> Faculty of Applied Physics and Mathematics -> Faculties of Gdańsk University of Technology		
Name and surname of lecturer (lecturers)	Subject supervisor	dr hab. Sergey Kryzhevich	
	Teachers	dr hab. Sergey Kryzhevich	

Lesson types

Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
Number of study hours	15.0	0.0	0.0	0.0	0.0	15

E-learning hours included: 0.0

eNauczanie source address: https://enauczanie.pg.edu.pl/2025/course/view.php?id=5576

Moodle ID: 5576 Wprowadzenie do teorii ergodycznej https://enauczanie.pg.edu.pl/2025/course/view.php?id=5576

Learning activity and number of study hours

Learning activity	Participation in didactic classes included in study plan	Participation in consultation hours	Self-study	SUM
Number of study hours	15	2.0	8.0	25

Subject objectives

To let students deal with the concepts of invariant measures, entropy, and information. To introduce them to the current state of the relevant theory and to discuss a number of issues at the intersection of the theory of random processes and dynamical systems.

Learning outcomes

Course outcome	Subject outcome	Method of verification
[K7_W101] is able to make an in-depth identification of key objects and phenomena related to the field of study, as well as theories that describe them and applicable analytical and design methods	Student has an understanding of the basic definitions, statements and methods of the theory of dynamic systems, topological and metric, and is able to apply them to solve specific problems.	[SW1] Assessment of factual knowledge
[K7_K101] acknowledges the importance of knowledge related to the field of study in solving cognitive and practical problems, critically assessing the information obtained	Students are expected to be able to apply their acquired knowledge to the study of practical problems. For example, they will apply information criteria to the study of random processes.	[SK2] Assessment of progress of work
[K7_U101] is able to formulate complex research problems and adopts appropriate methods, obtaining innovative solutions, cooperating with other people, both as a leader and a team member	Students are expected to understand the essence of the problems presented in modern scientific articles related to the field of study, as well as to master modern research methods, such as finding steady-state solutions to a system, estimating entropy, and so on.	[SU1] Assessment of task fulfilment [SU2] Assessment of ability to analyse information [SU4] Assessment of ability to use methods and tools

<table>
<tr><td>Subject contents</td><td>Course content – lecture
1. Dynamical systems, discrete and continuous.

2. Topological dynamics: periodic points, non-wandering points, recurrence.

3. Examples of dynamical systems: mappings of a segment, symbolical dynamics.

4. Invariant measures: definition and basic properties.

5. Existence of invariant measures, the Krylov-Bogolyubov Theorem.

6. Recurrence. The Poincaré Theorem.

7. Ergodic measures and their properties.

8. Entropy. Topologic and metric entropy.

9. Examples of calculating entropy. Information,

10. Random processes.</td></tr>
<tr><td>Prerequisites and co-requisites</td><td>It is supposed that the students have a basic knowledge in Analysis, Differential Equations, and Probability Theory. The knowledge in Statistics is desirable but not required.</td></tr>
<tr><td>Assessment methods and criteria</td><td><table><tr><th>Subject passing criteria</th><th>Passing threshold</th><th>Percentage of the final grade</th></tr><tr><td>examine</td><td>51.0%</td><td>100.0%</td></tr></table></td></tr>
<tr><td>Recommended reading</td><td>Basic literature</td><td>1. Leo Breiman, Probability. Original edition published by AddisonWesley, 1968; reprinted by Society for Industrial and Applied Mathematics, 1992. ISBN 0-89871-296-3. (See Chapter 6.)

2. Walters, Peter (1982), An introduction to ergodic theory, Graduate Texts in Mathematics, vol. 79, Springer-Verlag, ISBN 0-387-95152-0, Zbl 0475.28009 Bedford, Tim;

3. Karl Petersen. Ergodic Theory (Cambridge Studies in Advanced Mathematics). Cambridge: Cambridge University Press. 1990.

4. A. N. Shiryaev, Probability, 2nd ed., Springer 1996, Sec. V.3. ISBN 0-387-94549-0.

5. Andrzej Lasota, Michael C. Mackey, Chaos, Fractals, and Noise: Stochastic Aspects of Dynamics. Second Edition, Springer, 1994.</td></tr>
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	Supplementary literature	1. Vladimir Igorevich Arnol'd and André Avez, Ergodic Problems of Classical Mechanics. New York: W.A. Benjamin. 1968. 2. Keane, Michael; Series, Caroline, eds. (1991), Ergodic theory, symbolic dynamics and hyperbolic spaces, Oxford University Press, ISBN 0-19-853390-X (A survey of topics in ergodic theory; with exercises.) 3. Françoise Pène, Stochastic properties of dynamical systems, Cours spécialisés de la SMF, Volume 30, 2022 4. Manfred Einsiedler and Thomas Ward, Ergodic Theory with a view towards Number Theory. Springer, 2011. 5. Jane Hawkins, Ergodic Dynamics: From Basic Theory to Applications, Springer, 2021. ISBN 978-3-030-59242-4
	eResources addresses	
Example issues/ example questions/ tasks being completed	1. Calculate the topological entropy for the circle-doubling map. 2. Prove that the canonical invariant measure for the classical symbolic dynamics is ergodic. 3. Formulate the Sharkovskii theorem and prove it for the period-3 point.	
Practical activites within the subject	Not applicable	

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