GDAŃSK UNIVERSITY OF TECHNOLOGY

Subject card

Subject name and code	Differential equations II, PG_00070304		
Field of study	Mathematics		
Date of commencement of studies	October 2026	Academic year of realisation of subject	2026/2027
Education level	second-cycle studies	Subject group	Specialty subject group Subject group related to scientific research in the field of study
Mode of study	Full-time studies	Mode of delivery	at the university
Year of study	1	Language of instruction	Polish
Semester of study	2	ECTS credits	5.0
Learning profile	general academic profile	Assessment form	exam
Conducting unit	Department of Differential Equations and Mathematical Applications -> Faculty of Applied Physics and Mathematics -> Faculties of Gdańsk University of Technology		
Name and surname of lecturer (lecturers)	Subject supervisor	dr inż. Marcin Styborski	
	Teachers	dr inż. Marcin Styborski	

Lesson types

Lesson type	Lecture	Tutorial	Laboratory	Project	Seminar	SUM
Number of study hours	30.0	30.0	0.0	0.0	0.0	60
E-learning hours included: 0.0						

Learning activity and number of study hours

Learning activity	Participation in didactic classes included in study plan	Participation in consultation hours	Self-study	SUM
Number of study hours	60	5.0	60.0	125

Subject objectives

Acquiring basic knowledge about the qualitative theory of differential equations. Review of classical equations

Learning outcomes

Course outcome	Subject outcome	Method of verification
[K7_U07] at an advanced level and covering modern mathematics, applies and presents in speech and in writing the content and methods of a selected branch of mathematics	The student is able to analyze elementary nonlinear equations. Creates a phase portrait, examines the stability of solutions.	[SU4] Assessment of ability to use methods and tools
[K7_U03] uses differential and integral calculus, elements of complex analysis, algebraic methods, applies them in typical practical	The student is able to apply methods of calculus to analyze differential equations.	[SU4] Assessment of ability to use methods and tools
[K7_W05] demonstrates knowledge the numerical methods used to find approximate solutions to mathematical problems posed by applied fields	The student is able to use available software to analyze nonlinear equations.	[SW1] Assessment of factual knowledge
[K7_W01] has enhanced knowledge of basic branches of mathematics,demonstrates knowledge theorem and hypotheses, has understanding of the role and importance of mathematical reasoning structure.	The student is able to apply the methods of linear algebra, mathematical analysis and topology to the qualitative study of differential equations.	[SW1] Assessment of factual knowledge

<table>
<tr><td>Subject contents</td><td colspan="2">Course content – lecture
1. Introduction. What are dynamical systems and why do they matter? Flow, orbits, phase portrait. Course overview. Assessment criteria.
2. Planar classification of linear systems (TrDet diagram). Topological conjugacy.
3. Topological classification in R^n. Hyperbolic and non-hyperbolic cases. Topological conjugacy vs. matrix similarity.
4. Stable, unstable, and center manifolds. Hyperbolic fixed points. Stable and unstable manifold theorem. Example: Duffing equation.
5. Straightening-out Theorem. Regular points vs. equilibria. Local structure of trajectories. Poincaré section introduction.
6. Lyapunov stability. Definitions: stability, asymptotic stability, instability. Lyapunov stability theorem.
7. Lyapunov functions construction methods. Instability theorem. Estimation of basins of attraction. Examples: harmonic oscillator, pendulum.
8. Applications of the Lyapunov method. Gradient systems. Energy as a Lyapunov function. Examples: van der Pol oscillator, Lorenz system.
9. Center manifold. Existence and regularity theorem. Dimension reduction. Dynamics on the center manifold.
10. Hamiltonian systems and first integrals. Conservation of energy. Examples: pendulum, Duffing. Liouville's theorem on measure.
11. PoincaréBendixson theorem part I. Omega-limit sets. Limit cycles.
12. PoincaréBendixson theorem part II. Criteria excluding cycles: Bendixson, Dulac. Applications: population models (LotkaVolterra), van der Pol.
13. Poincaré map. Periodic orbits and their stability. PoincaréHopf index theorem.
14. Deterministic chaos. Lorenz attractor. Sensitivity to initial conditions.

Course content – exercises
During the exercises, students receive worksheets on which they work in groups. Each worksheet covers a topic covered in the lecture.</td></tr>
<tr><td>Prerequisites and co-requisites</td><td colspan="2">The knowledge of Mathematical Analysis, Differential Equations I</td></tr>
<tr><td>Assessment methods and criteria</td><td colspan="2">
<table>
<tr><th>Subject passing criteria</th><th>Passing threshold</th><th>Percentage of the final grade</th></tr>
<tr><td>worksheet</td><td>50.0%</td><td>90.0%</td></tr>
<tr><td>exam</td><td>50.0%</td><td>10.0%</td></tr>
</table></td></tr>
<tr><td rowspan="3">Recommended reading</td><td>Basic literature</td><td>1. M. W. Hirsch, S. Smale, R. L. Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos, Academic Press (Elsevier), wyd. 2 / 2nd ed., 2004. Główny podręcznik kursu. Geometryczna intuicja, klasyfikacja układów liniowych, twierdzenie Poincarégo-Bendixsona, van der Pol, Lorenz.
2. L. Perko, Differential Equations and Dynamical Systems, Springer, wyd. 3 / 3rd ed., 2001. (Texts in Applied Mathematics, vol. 7) Precyzja i kompletność. Rozmaitości stabilne (§2.7), stabilność Lapunowa (§2.9), rozmaitość centralna (§2.12), układy hamiltonowskie (§2.14).
3. G. Teschl Ordinary Differential Equations and Dynamical Systems, American Mathematical Society, 2012.</td></tr>
<tr><td>Supplementary literature</td><td></td></tr>
<tr><td>eResources addresses</td><td></td></tr>
<tr><td>Example issues/ example questions/ tasks being completed</td><td colspan="2">Exercise 1.Investigate the stability of the equilibrium point of the system using a Lyapunov function

Exercise 2.For the system find all equilibrium points, classify them via linearization, and sketch the phase portrait.</td></tr>
<tr><td>Practical activites within the subject</td><td colspan="2">Not applicable</td></tr>
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