GDAŃSK UNIVERSITY OF TECHNOLOGY

Subject card

<table>
<tr><td>Subject name and code</td><td colspan="3">AI-supported learning – matrix decompositions, PG_00072601</td></tr>
<tr><td>Field of study</td><td colspan="3">Technical Physics, Materials Engineering, Mathematics, Materials Engineering, Nanotechnology, Nanotechnology, Materials Engineering</td></tr>
<tr><td>Date of commencement of studies</td><td>October 2025</td><td>Academic year of realisation of subject</td><td>2026/2027</td></tr>
<tr><td>Education level</td><td>second-cycle studies</td><td>Subject group</td><td>Optional subject group</td></tr>
<tr><td>Mode of study</td><td>Full-time studies</td><td>Mode of delivery</td><td>at the university</td></tr>
<tr><td>Year of study</td><td>2</td><td>Language of instruction</td><td>Polish</td></tr>
<tr><td>Semester of study</td><td>3</td><td>ECTS credits</td><td>1.0</td></tr>
<tr><td>Learning profile</td><td>general academic profile</td><td>Assessment form</td><td>assessment</td></tr>
<tr><td>Conducting unit</td><td colspan="3">Institute of Applied Mathematics -> Faculty of Applied Physics and Mathematics -> Faculties of Gdańsk University of Technology</td></tr>
<tr><td rowspan="2">Name and surname of lecturer (lecturers)</td><td>Subject supervisor</td><td colspan="2">dr hab. Karol Dziedziul</td></tr>
<tr><td>Teachers</td><td colspan="2">dr hab. Karol Dziedziul</td></tr>
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<tr><td rowspan="3">Lesson types</td><td>Lesson type</td><td>Lecture</td><td>Tutorial</td><td>Laboratory</td><td>Project</td><td>Seminar</td><td>SUM</td></tr>
<tr><td>Number of study hours</td><td>0.0</td><td>0.0</td><td>15.0</td><td>0.0</td><td>0.0</td><td>15</td></tr>
<tr><td colspan="7">E-learning hours included: 0.0</td></tr>
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<tr><td rowspan="2">Learning activity and number of study hours</td><td>Learning activity</td><td>Participation in didactic classes included in study plan</td><td>Participation in consultation hours</td><td>Self-study</td><td>SUM</td></tr>
<tr><td>Number of study hours</td><td>15</td><td>2.0</td><td>8.0</td><td>25</td></tr>
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<tr><td>Subject objectives</td><td>Students will be introduced to selected matrix decomposition methods, including the singular valence decomposition (SVD), the Cholesky decomposition, and the CUR. Practical use of ChatGPT in mathematics learning: generating solutions, analyzing results, and assessing the correctness of answers. Students will develop independent mathematical problem-solving skills with the support of artificial intelligence, with an emphasis on critical thinking and assessing the quality of answers.</td></tr>
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Learning outcomes

Course outcome	Subject outcome	Method of verification
[K7_W101] is able to make an in-depth identification of key objects and phenomena related to the field of study, as well as theories that describe them and applicable analytical and design methods	The student identifies, at an advanced level, key objects and phenomena related to matrix decompositions, in particular matrices, eigenvalues and eigenvectors, singular values, matrix rank, projections, and low-rank approximations. The student knows the basic theories describing these concepts and is able to indicate applicable analytical and design methods, including SVD, PCA, and other selected matrix decompositions. The student is able to use these methods, with the support of AI tools, for data analysis and image processing, as well as to critically assess the correctness of the obtained results.	[SW2] Assessment of knowledge contained in presentation
[K7_U101] is able to formulate complex research problems and adopts appropriate methods, obtaining innovative solutions, cooperating with other people, both as a leader and a team member	The student formulates complex research problems related to matrix decompositions and selects appropriate analytical and computational methods for solving them. Using AI tools as support in analysis, literature exploration, hypothesis generation, and solution verification, the student develops original concepts for applying matrix decompositions, particularly in data analysis and image processing. The student is able to collaborate with others in solving problems, both as a leader and as a member of a team.	[SU5] Assessment of ability to present the results of task
[K7_K101] acknowledges the importance of knowledge related to the field of study in solving cognitive and practical problems, critically assessing the information obtained	The student recognises the importance of knowledge related to matrix decompositions and their applications in solving cognitive and practical problems, particularly in data analysis and image processing. The student is able to deepen their own understanding of the discussed topics by using AI tools as support in the learning process. At the same time, the student critically evaluates information obtained from AI, verifies its correctness using mathematical knowledge and analytical methods, and recognises the limitations of automatically generated responses.	[SK2] Assessment of progress of work

Subject contents

Course content – laboratory

Basic matrix concepts. More-Penrose matrices. Matrix decompositions: Cholesky, LU, Rank factorization, CUR, singular SVD, Jordan, Schur, QZ.
Additional refreshers or supplementary concepts: Rank of a matrix, Uniqueness theorem, Inverse matrix (when $\det(A)\neq 0$), Projection of a vector onto a subspace, Determinant of a matrix and its geometric interpretation, Matrix as operator $A:R^n \to R^m$, operator image and its kernel, Complex matrix as operator $A:C^n \to C^m$, operator image and its kernel, Orthogonal matrix Q: $Q^\top Q = I$, Vandermonde matrix and interpolation with polynomials and their relationship, Orthogonal matrices, example: Hadamar matrix.
Discussion of prompt formulation methods that allow for obtaining accurate results.

Prerequisites and co-requisites

Create an account at https://chatgpt.com. In principle, a course in linear algebra is not required. The limited knowledge of algebra taught in universities can even be limiting.

Assessment methods and criteria

Subject passing criteria	Passing threshold	Percentage of the final grade
Mini-tests at the end of each block	60.0%	100.0%

Recommended reading	Basic literature	Gilbert Strang Linear Algebra nad its Application Gilbert Strang The geometry of linear equations (MIT OCW)
	Supplementary literature	Lloyd N. Trefethen & David Bau III "Numerical Linear Algebra Ivan Markovsky "Low Rank Approximation: Algorithms, Implementation, Applications"
	eResources addresses	Basic https://mostwiedzy.pl/pl/karol-dziedziul,4112-1/rozklady-macierzowe - see most wiedzy dziedziul. rozkłady macierzowe
Example issues/ example questions/ tasks being completed	1. Compute the singular value decomposition (SVD) for a matrix A=(1324). 2. Compute the Cholesky decomposition for a symmetric matrix, e.g., A=(4223). 3. Explain when the CUR decomposition is used, giving an example of its application in data analysis. 4. For any square matrix \$B\$ of dimension \$3\times 3\$, find the matrix \$P_I\$ that eliminates the second raw, i.e., for \$I=\{1,3\}\$ \[P_IB=B(I,:) \] for any matrix \$B\$ of dimension \$3\times 4\$. 5. Compare the quality of solutions (e.g., reconstruction error) in NMF for different starting points	
Practical activites within the subject	Not applicable	